

Interaction of Wilson loops in confining vacuum

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Abstract

Nonperturbative and perturbative interaction mechanisms of Wilson loops are studied within the background field formalism. The first one operates when distance between minimal surfaces of the loops is small and may be important for sea quark effects and strong decay processes. The second mechanism – perturbative interaction in nonperturbative confining background is found to be physically dominant for all loop configurations characteristic of scattering process. It reduces to perturbative gluon exchanges at small distances, while at larger distances it corresponds to the t -channel exchange of glueball states. Comparison to other approaches is made and possible physical applications are discussed.

1 Introduction

Interaction of Wilson loops in QCD is the basic element of many physical applications. One can mention hadron-hadron scattering amplitude, in particular the phenomenology of Pomeron exchange; the assumed color transparency phenomenon; strong hadron decays and OZI-forbidden processes etc. There is also considerable interest in calculating Wilson loop and Polyakov loop correlators *per se*, not only in QCD but also in other field theories, in particular in supersymmetric Yang-Mills theory. In all cases one starts with connected average of two (or more) Wilson loops and tries to calculate it in a kinematic region of interest using appropriate field-theoretical technique. It is the aim of the present paper to do it in the framework of field correlator method in gluodynamics (see, e.g. review [1] and references therein) incorporating both perturbative and nonperturbative contributions. Let us briefly remind the basic ideas behind the method. The general Wilson loop approach was introduced originally for heavy quarkonia [2]. For quarks of finite mass one can use Feynman-Schwinger representation (see [3] for a review and references therein) to write the meson Green's function as an integral over all possible Wilson loops, formed by the quark trajectories and finally to express the meson (and baryon) dynamics in terms of gauge-invariant correlators of the field strengths, characterizing the properties of confining background. When going to the hadron-hadron scattering one can adopt the same formalism to express scattering amplitude through the vacuum average of the product of two Wilson loops, with subsequent integration over all ensemble of loops. In doing so we use the background field formalism to separate nonperturbative gluon configurations from perturbative (sometimes called "valence") gluons. Thus the answer will contain two parts: purely nonperturbative and perturbative inside nonperturbative background, i.e. glueball exchanges between Wilson loops.

For the former it is convenient to use nonabelian Stokes theorem and express the answer in terms of the gauge-invariant field correlators and finally via the string tension. Perturbative part in the nonperturbative background corresponds to exchange of glueball states between loops. We shall keep number of colors N_c as a free parameter in what follows. It will be argued that the leading term for a typical kinematics of the scattering process is the (background-modified) perturbative one. This is in line with the old observation that high-energy scattering amplitude is dominated by the Pomeron exchange. We do not consider here the leading in $1/N_c$ terms of ordinary Reggeon exchanges, which formally refer to one-loop case and are subleading in the high energy limit. We are also not discussing pion exchanges which may give the main contribution in some cases at not large energies, and will concentrate our attention on the case of theory without dynamical quarks, i.e. gluodynamics. In another physical situation, e.g. when accounting for the sea-quark loops or for a decay transition of a hadron state, the roles of perturbative and nonperturbative mechanisms may change depending on the hadron quantum numbers. In all cases however the loop-loop interaction is the starting point of field-correlator formalism application for scattering, strong decay etc. In section 2 we introduce the general background formalism for the interaction of Wilson loops. In section 3 the nonperturba-

tive mechanism is studied in detail. In section 4 perturbative gluon exchange is shown to transform into glueball exchange mechanism at large distances. Section 5 is devoted to a physical discussion of results and brief comparison with the existing models.

2 Interaction of Wilson loops in the background field formalism

In this section we are going to exploit the background field formalism [4] in the form worked out in [5]. We refer the interested reader to the cited papers for all the details and recall basic steps only briefly. We start with decomposing of gluon field $A_\mu(x)$ into nonperturbative background $B_\mu(x)$ and perturbative part a_μ , propagating in the background:

$$A_\mu = B_\mu + a_\mu \quad (1)$$

Total gauge transformation is decomposed as

$$B_\mu \rightarrow U^\dagger \left(B_\mu - \frac{i}{g} \partial_\mu \right) U \quad ; \quad a_\mu \rightarrow U^\dagger a_\mu U \quad (2)$$

The principle of separation is of no importance at the level of partition function due to obvious identity

$$Z = \frac{1}{\mathcal{N}} \int \mathcal{D}A_\mu \exp(-S[A]) = \frac{1}{\mathcal{N}'} \int \mathcal{D}B_\mu \int \mathcal{D}a_\mu \exp(-S[B + a]) \quad (3)$$

(here gauge-fixing and ghost terms are assumed to be included into the measure of integration). The Wilson loop depends on both B_μ and a_μ :

$$\begin{aligned} W(C) &= \frac{1}{N_c} \text{Tr} \text{P exp} \left(ig \int_C (B_\mu^c + a_\mu^c) t^c dz_\mu \right) = \\ &= \frac{1}{N_c} \lim_{M \rightarrow \infty} \text{Tr} \prod_{m=1}^M \text{P} \left(1 + ig(B_\mu(z^{[m]}) + a_\mu(z^{[m]})) \Delta z_\mu^{[m]} \right) \end{aligned} \quad (4)$$

The trace in fundamental representation is normalized as

$$\text{Tr} \hat{\mathbf{1}} = N_c \quad ; \quad \text{Tr} t^a t^b = \frac{1}{2} \delta^{ab}$$

Our general strategy is the following [5]: we expand the correlators under study in powers of the field a_μ , while account for effects caused by the nonperturbative background exactly (i.e. without expansion in powers of B_μ). Namely, one has

$$W(C) = W^{(0)}(C) + W^{(1)}(C) + W^{(2)}(C) + \dots \quad (5)$$

where, e.g.

$$W^{(1)}(C) = \frac{ig}{N_c} \text{Tr} P_{zu} \int_C a_\mu(z) dz_\mu \exp \left(ig \int_{C_z} B_\nu(u) du_\nu \right) \quad (6)$$

while $W^{(0)}(C)$ contains only the field B_μ and ordering operator P_{zu} takes care of ordering $a_\mu(z)$ and $B_\mu(u)$.

Let us define now connected average of two Wilson loops as

$$\chi(C_1, C_2) = \langle W(C_1)W(C_2) \rangle - \langle W(C_1) \rangle \langle W(C_2) \rangle \quad (7)$$

This average can also be expanded in powers of ga_μ :

$$\chi(C_1, C_2) = \chi^{(0)}(C_1, C_2) + \chi^{(2)}(C_1, C_2) + \chi^{(4)}(C_1, C_2) + \dots \quad (8)$$

Here $\chi^{(0)}(C_1, C_2)$ is purely nonperturbative interaction of two Wilson loops and depends only on fields B_μ , while higher terms $\chi^{(n)}(C_1, C_2)$ are proportional to the average of $(ga_\mu)^n$. One immediately notices that since $\langle W^{(1)} \rangle$ is identically zero, the term $\chi^{(2)}(C_1, C_2)$ vanishes and the expansion starts with the two-gluon exchange term $\chi^{(4)}(C_1, C_2)$.

In some cases also C -odd exchange contribution (odderon-type) is important, it is contained in $\chi^{(6)}(C_1, C_2)$. In what follows we discuss mostly the purely nonperturbative term $\chi^{(0)}(C_1, C_2)$, and in the last part of the paper also two-gluon exchange $\chi^{(4)}(C_1, C_2)$ in section 4.

3 Nonperturbative interaction of Wilson loops

We consider in this section the first term in the expansion (8), namely $\chi^{(0)}(C_1, C_2)$ and use the contour gauge [6] to write down the Wilson loop as surface integral

$$W(C) = \frac{1}{N_c} \text{Tr} P \exp \left(ig \int_S F_{\mu\nu}(u, x_0) d\sigma_{\mu\nu}(u) \right) \quad (9)$$

We have defined in (9) $F_{\mu\nu}(u, x_0) = \Phi_{L_{x_0u}} F_{\mu\nu}(u) \Phi_{L_{ux_0}}$ where phase factors along the curve L_{x_0u} with the edge points x_0 and u are given by

$$\Phi_{L_{x_0u}} = P \exp \left(ig \int_u^{x_0} B_\mu(z) dz_\mu \right) \quad (10)$$

Consider Wilson loops defined for two contours C_1 and C_2 , where individual minimal surfaces will be denoted as S_1^{min} and S_2^{min} , respectively throughout the paper.¹ The typical problem in the discussed framework is to choose optimal integration surfaces in the

¹For any surface we use one and the same letter S for a surface as geometrical object and for its area.

integrals of the form (9). The Wilson loop (9) is gauge-invariant and surface-independent. In the confining regime it obeys the area law, which means that a single surface is unambiguously chosen dynamically (modulo surface fluctuations) and it is naturally assumed to be the minimal surface. In the approach proposed in [7] one can visualize how the minimal surface results from the balance of action of all correlators, and higher correlators play important role in making the surface smooth and minimal. After this goal is achieved, the relative importance of correlators on the minimal surface obeys a strict hierarchy. For a single Wilson loop it can be strongly argued (see discussion and references in [1], [8]), that in case of minimal surface (which is obviously the distinguished surface for given contour) the dominant nonperturbative contribution in the cluster expansion of (9) comes from the lowest, Gaussian correlator of the field strength operators. This property is known as Gaussian dominance [7] and it plays important role in all phenomenological applications of field correlator method.

In the physical picture described above appearance of the minimal surface has been in some sense a result of field correlator dynamics. However, one can take another view by saying that Gaussian dominance corresponds to such profile of the confining string worldsheet, which minimizes the total energy of the system. In other words, to calculate $\langle W(C) \rangle$ one may proceed as follows: first, to find the minimal energy of confining string configuration (trivially corresponding to minimal area surface in case of single static loop) and, as the next step, to calculate average (9) with Gaussian ensemble of correlators integrated over this surface. It is this principle which we shall use in what follows to choose the shape of the surfaces entering our problem. To illustrate the physics behind it, let us consider gauge-invariant Green's function of scalar particle of the mass M_1 and antiparticle of the mass M_2 interacting with nonabelian gauge field A_μ^a (see, e.g. [3])

$$G^{[2]}(x, y) = \langle \phi^\dagger(x) \Phi_{L_{x\bar{x}}} \phi(\bar{x}) \phi^\dagger(\bar{y}) \Phi_{L_{\bar{y}y}} \phi(y) \rangle =$$

$$= \int_0^\infty ds_1 \int_0^\infty ds_2 \int (\mathcal{D}z)_{xy} \int (\mathcal{D}\bar{z})_{\bar{x}\bar{y}} \exp(-K_0) \langle W(C_{xy\bar{y}\bar{x}}) \rangle \quad (11)$$

where

$$K_0 = M_1^2 s_1 + M_2^2 s_2 + \frac{1}{4} \int_0^{s_1} d\tau_1 \left(\frac{dz_\mu}{d\tau_1} \right)^2 + \frac{1}{4} \int_0^{s_2} d\tau_2 \left(\frac{d\bar{z}_\mu}{d\tau_2} \right)^2 \quad (12)$$

and the contour $C_{xy\bar{y}\bar{x}}$ is formed by dynamical trajectories z, \bar{z} and the lines $L_{x\bar{x}}, L_{\bar{y}y}$. Green's function (11) encodes all information about the spectrum of the system. Of special interest are particular cases when some trajectories in the integrals in (11) are singled out kinematically as providing the main contribution. Phenomenologically the most important cases are represented by the large mass (nonrelativistic) limit, where the particle trajectories are close to parallel to the temporal (Euclidean) axis, and large-momentum limit, where eikonal approximation is applicable. Let us consider the former case in more details and assume for simplicity that $x_4 = \bar{x}_4$, $y_4 = \bar{y}_4$, $y_4 - x_4 = T$, and center-of-mass motion of the considered two-body system is absent. The Green's function

(11) can be spectrally decomposed as

$$G^{[2]}(x, \bar{x}; y, \bar{y}) = \sum_{\{n\}} \psi_{\{n\}}^\dagger(\vec{x} - \vec{\bar{x}}) \psi_{\{n\}}(\vec{y} - \vec{\bar{y}}) \exp(-E^{\{n\}}T) \quad (13)$$

where T stays for the Euclidean time interval while $\psi_{\{n\}}, E^{\{n\}}$ denote wave function and energy of the state with quantum numbers $\{n\}$, respectively. Comparing (11) and (13) we notice that the problem of minimizing energy levels is in close correspondence with the problem of maximizing average value of the Wilson loop $\langle W(C) \rangle$ for given contour, prescribed by kinematics. This correspondence becomes straightforward in classical limit of static rectangular loop where area law dictates

$$\langle W(R \times T) \rangle \approx \exp(-ET)$$

and the static energy is given by $E = \sigma R$, which corresponds to straight line string and minimal area. Notice that we confine ourselves to the leading area law terms in this paper and do not consider perimeter contributions.

For two or more Wilson loops the situation is more complicated, but general framework is the same. Indeed, one can consider multi-point Green's functions, e.g. two-meson one

$$G^{[4]} = \langle \Psi_{Lx\bar{x}}^\dagger \Psi_{Lu\bar{u}}^\dagger \Psi_{Ly\bar{y}} \Psi_{Lv\bar{v}} \rangle \quad (14)$$

where

$$\Psi_{Ly\bar{y}} = \phi^\dagger(y) \Phi_{Ly\bar{y}} \phi(\bar{y})$$

and again adopt Feynman-Schwinger representation to get

$$G^{[4]} \sim \int [* * *] \cdot \langle W(C_1) W(C_2) \rangle + \dots \quad (15)$$

where stars $[* * *]$ stay for integration measure while dots denote other possible ways to contract field operators in the definition (14) and get, correspondingly, other geometries of contours C_1, C_2 . On the other hand, the Green's function (14) describing interacting two-meson system, must be dominated at large separation/time by the confining strings configuration of lowest total energy, in direct analogy with one meson case.

It is physically obvious that for two well separated loops the minimal configuration of confining strings is given by two individual minimal surfaces with no common points of intersection. From the point of view of the expression (15) it means that average $\langle W(C_1) W(C_2) \rangle$ factorizes into $\langle W(C_1) \rangle \langle W(C_2) \rangle$ and no nonperturbative interaction takes place. Notice that perturbative interaction is mediated in our picture by propagating degrees of freedom, while $\chi^{(0)}$ could receive nonzero contributions only because of rearrangement of confining strings with respect to the free case, if it happens. In the approach developed in the present paper it is always assumed that the gluon correlation length T_g (which also defines the radius of the string) is taken vanishingly small, while the string tension σ is kept fixed and therefore $\chi^{(0)}$ is expressed in terms of σ alone. This is in contrast to another approach [9], to be discussed later in the paper.

Consider now a process of decreasing the distance between the loops. For classical films, as it is easily checked in experiments with soap films, at some point there appears the situation, when the minimal surface, bounded by contours C_1 and C_2 , which we shall call S_{12} , becomes smaller, than the sum of S_1^{min} and S_2^{min} . At this point, according to minimum energy principle, there should appear a common surface S_{12} which constitutes the interaction between the loops. Indeed the classical films (e.g. soap films) satisfy the stated above principle. For the simplest case of two concentric circles of equal radius R at distance h it was Leonard Euler who demonstrated that the corresponding minimal surface S_{12} is a catenoid and it exists for any h such that the equation

$$\frac{R}{R_{min}} = \cosh\left(\frac{h}{2R_{min}}\right) \quad (16)$$

has a solution. Here R_{min} defines the so called minimal radius of the catenoid. At some critical distance h_{cr} the solution ceases to exist and the minimal surface coincides with two disconnected disks.²

The case of QCD is more complicated, first of all because of nonabelian nature of the gluon field. In particular, it is possible strictly speaking to take in (9) integration surface with the topology different from that of the disc only at the price of introducing additional holonomy factors.³ It is also worth mentioning that the surface corresponding to the global minimum of energy can be glued with itself in nontrivial way, as it happens, for example, in baryon correlators. In fact, soap films demonstrate a similar behavior to some extent: they easily form "string junction" with three worldsheets having one common intersection line and forming e.g. "Mercedes-star" configurations. Moreover, contrary to the case of one Wilson loop there is no, in some sense, a unique distinguished string worldsheet configuration for the product of two or more loops. Let us illustrate this on the example of two coinciding and oppositely oriented Wilson loops, where the following well known identity holds true

$$|W_{fund}|^2 = \frac{1}{N_c^2} + \left(1 - \frac{1}{N_c^2}\right) \cdot W_{adj} \quad (17)$$

where W_{fund} is given just by (4), while W_{adj} corresponds to (4) after replacement of fundamental $SU(N_c)$ generators t^a with generators $[T^a]_{bc} = -if^{abc}$ in adjoint representation and proper change of trace normalization.⁴ The terms in the r.h.s. of (17) are in one-to-one correspondence with decomposition of the product of two fundamental $SU(N_c)$ representations into singlet and adjoint: $N_c \otimes \bar{N}_c = \mathbf{1} \oplus (N_c^2 - 1)$. Accordingly, the answer

²Actually the analog of condition (42) is violated earlier, at some distance $\tilde{h}_{cr} < h_{cr}$. The catenoid solution is locally stable but globally unstable for $\tilde{h}_{cr} < h < h_{cr}$. At $h > h_{cr}$ local stability is lost. We do not make a distinction between \tilde{h}_{cr} and h_{cr} in our consideration and speaking about critical distance we always have in mind minimal energy condition.

³See, e.g. [10]. It is in contrast with abelian Stokes theorem where one can easily integrate two-form field strength over noncontractible surface.

⁴Traces (both fundamental and adjoint) are normalized to unity in (17).

is given by superposition of the state where two fundamental confining strings are totally annihilated (corresponding to the unit representation and constant term in (17)), and the state where confining strings are summed into one adjoint string (described by the second term in the r.h.s. of (17)). It is clear that for large contours the second contribution dies out and the minimal energy state is given by the singlet state.

Till the present moment we have discussed minimization of energies, paying no attention to pre-exponential factors like $1/N_c^2$ in (17). The latter can become important in studies of large- N_c behavior of the correlators. We shall come back to this question later on.

The physical picture outlined above is in fact quite general. Therefore we adopt the following algorithm for computing nonperturbative Wilson loop correlator. First, for given geometry of the contours, we have to find confining string configuration which gives dominant contribution to the quantity $\langle W(C_1)W(C_2) \rangle$ (notice, that there might be different surfaces for different representations, which appear in the expansion of the product of fundamental Wilson loops, as in (17)). If the configuration, providing the maximum coincides with individual minimal surfaces, we conclude that nonperturbative interaction is absent. As the second step, we calculate correlator $\chi^{(0)}(C_1, C_2)$ via (9) in terms of gauge-invariant field correlators. We assume that the ensemble of correlators obeys the same hierarchy on this surface as it does on the minimal surface for single Wilson loop, i.e. exhibits Gaussian dominance. The strategy adopted below for computation of higher terms in the expansion (8) is slightly different but basic physical arguments for choosing minimal surfaces remain the same (see section 4).

We are keeping generality at the moment and perform the vacuum averaging of Wilson loops, i.e. we explicitly calculate $\chi^{(0)}(C_1, C_2)$. Suppose that surfaces S_1 , S_2 have already been chosen according to our criteria. For field-strength tensors, belonging to surfaces S_1 and S_2 and gauge-transported to the same point x_0 we define:

$$\begin{aligned} (Fd\sigma)^{(1)}(u) &= F_{\mu\nu}(u, x_0)d\sigma_{\mu\nu}(u) \ ; \ u \in S_1 \\ (Fd\sigma)^{(2)}(v) &= F_{\mu\nu}(v, x_0)d\sigma_{\mu\nu}(v) \ ; \ v \in S_2 \end{aligned} \quad (18)$$

One can now write down the product of two Wilson loops in matrix form as

$$\begin{aligned} N_c^2 \langle W(C_1)W(C_2) \rangle &= \\ &= \left\langle \text{Tr}_1 \text{Tr}_2 P_{12} \left[\exp \left(ig \int_{S_1} (Fd\sigma)^{(1)}(u) \right) \right]_{\alpha_1 \beta_1} \left[\exp \left(ig \int_{S_2} (Fd\sigma)^{(2)}(v) \right) \right]_{\alpha_2 \beta_2} \right\rangle \end{aligned} \quad (19)$$

where the traces Tr_1 and Tr_2 go over indices carrying subscripts 1 and 2 respectively and the ordering operator P_{12} orders the products of matrices in a proper way according to the definition of P-exponent (4).

One can derive the following rule for vacuum averaging of several matrix operators, transported to one point (it is easy to show that these relations are gauge-invariant under field-independent gauge transformations)

$$\begin{aligned}
& \langle [F(u^{(1)}, x_0) \dots F(u^{(k)}, x_0)]_{\alpha_1 \beta_1} [F(v^{(1)}, x_0) \dots F(v^{(m)}, x_0)]_{\alpha_2 \beta_2} \rangle = \\
& = \frac{\delta_{\alpha_1 \beta_2} \delta_{\alpha_2 \beta_1}}{N_c^2 - 1} \left[\langle \text{Tr} (F(u^{(1)}, x_0) \dots F(u^{(k)}, x_0) F(v^{(1)}, x_0) \dots F(v^{(m)}, x_0)) \rangle \right. \\
& \quad \left. - \frac{1}{N_c} \langle \text{Tr} (F(u^{(1)}, x_0) \dots F(u^{(k)}, x_0)) \text{Tr} (F(v^{(1)}, x_0) \dots F(v^{(m)}, x_0)) \rangle \right] + \\
& \quad \frac{\delta_{\alpha_1 \beta_1} \delta_{\alpha_2 \beta_2}}{N_c^2 - 1} \left[\langle \text{Tr} (F(u^{(1)}, x_0) \dots F(u^{(k)}, x_0)) \text{Tr} (F(v^{(1)}, x_0) \dots F(v^{(m)}, x_0)) \rangle \right. \\
& \quad \left. - \frac{1}{N_c} \langle \text{Tr} (F(u^{(1)}, x_0) \dots F(u^{(k)}, x_0) F(v^{(1)}, x_0) \dots F(v^{(m)}, x_0)) \rangle \right] \quad (20)
\end{aligned}$$

For lowest Gaussian correlator when $k = m = 1$ one has from (20)

$$\langle [F(u, x_0)]_{\alpha_1 \beta_1} [F(v, x_0)]_{\alpha_2 \beta_2} \rangle = \frac{\langle \text{Tr} F(u, x_0) F(v, x_0) \rangle}{N_c^2 - 1} \left(\delta_{\alpha_1 \beta_2} \delta_{\alpha_2 \beta_1} - \frac{1}{N_c} \delta_{\alpha_1 \beta_1} \delta_{\alpha_2 \beta_2} \right) \quad (21)$$

The above relations are valid in theories without global color symmetry breaking. Now one can proceed with matrix cumulant expansion (see, e.g. [11]):

$$N_c^2 \langle W(C_1) W(C_2) \rangle = \text{Tr}_1 \text{Tr}_2 P_{12} \exp \left(\sum_{n=1}^{\infty} \frac{(ig)^n}{n!} \langle \langle \tilde{F}(1) \dots \tilde{F}(n) \rangle \rangle \right) \quad (22)$$

where we use the notation

$$\tilde{F}(k) = \int_{S_1} (F d\sigma)^{(1)}(u^{(k)}) + \int_{S_2} (F d\sigma)^{(2)}(v^{(k)}) \quad (23)$$

Here double brackets $\langle \langle \dots \rangle \rangle$ denote irreducible correlators (see definition in [1, 11]). The quantity $\tilde{F}(k)$ carries four independent fundamenatal color indices and the traces Tr_1, Tr_2 go over indices corresponding to $F^{(1)}$ and $F^{(2)}$, respectively.

Expression (22) provides the basis for our discussion. For a single loop the corresponding cluster expansion is given by

$$\langle W(C_1) \rangle = \frac{1}{N_c} \text{Tr}_1 P \exp \left(\sum_{n=1}^{\infty} \frac{(ig)^n}{n!} \langle \langle \int_{S_1} (F d\sigma)^{(1)}(u^{(1)}) \cdot \dots \cdot \int_{S_1} (F d\sigma)^{(1)}(u^{(n)}) \rangle \rangle \right) \quad (24)$$

and in area law regime (which means that typical sizes of the loops are larger than the gluon correlation length T_g) one gets

$$\langle W(C_1) \rangle \propto \exp(-\sigma S_1^{min}) \quad (25)$$

The string tension σ is given by

$$\sigma_{S_1}^{min} = \frac{1}{2N_c} \int_{S_1} d\sigma(u^{(1)}) \int_{S_1} d\sigma(u^{(2)}) \langle \text{Tr } gF(u^{(1)}, x_0) gF(u^{(2)}, x_0) \rangle + \dots \quad (26)$$

where the dots denote higher non-Gaussian terms. As it was mentioned above, we do not take into account perimeter terms, it is implicitly supposed that all considered loops are large enough in this sense. It is straightforward to rewrite (22) combined with (24) and (20) in the following way

$$N_c^2 \langle W(C_1) W(C_2) \rangle = \text{Tr } {}_1 \text{Tr } {}_2 \exp \left(\hat{1} \cdot (\Lambda_0 + \Lambda_1) + \hat{e} \cdot \Lambda_e \right) \quad (27)$$

where

$$\Lambda_0 = -\sigma S_1 - \sigma S_2 \quad (28)$$

and terms Λ_1 and Λ_e contain correlators of powers of F defined on different surfaces and hence provide contribution to $\chi^{(0)}$. They are as follows:

$$\Lambda_1 = \pm \frac{g^2}{N_c(N_c^2 - 1)} \int_{S_1} d\sigma(u^{(1)}) \int_{S_2} d\sigma(v^{(1)}) \langle \text{Tr } F(u^{(1)}, x_0) F(v^{(1)}, x_0) \rangle + \dots \quad (29)$$

and

$$\Lambda_e = \mp \frac{g^2}{N_c^2 - 1} \int_{S_1} d\sigma(u^{(1)}) \int_{S_2} d\sigma(v^{(1)}) \langle \text{Tr } F(u^{(1)}, x_0) F(v^{(1)}, x_0) \rangle + \dots \quad (30)$$

where the dots again denote higher non-Gaussian terms. The upper (lower) sign in the above expressions corresponds to the case of parallel (opposite) orientation of the contours C_1 and C_2 and surfaces S_1 and S_2 (we assume that the orientation of the surface is fixed by the orientation of its boundary contour).

The matrix structures $\hat{1}$ and \hat{e} introduced above are given in index notation by

$$[\hat{1}]_{\alpha_1\beta_1;\alpha_2\beta_2} = \delta_{\alpha_1\beta_1} \delta_{\alpha_2\beta_2} \quad ; \quad [\hat{e}]_{\alpha_1\beta_1;\alpha_2\beta_2} = \delta_{\alpha_1\beta_2} \delta_{\alpha_2\beta_1} \quad (31)$$

We are to examine what algebra the matrices $\hat{1}$ and \hat{e} do obey. In the chosen approximation it encodes the effect of ordering, performed by the operator P_{12} . Consider the case of parallel orientation. In this case ordering of the fields $F^{(1,2)}$ in the Wilson loop product coincide for both loops, which for the matrices $\hat{1}$ and \hat{e} mean, that they should be multiplied from the right with respect to indices carrying subscript "1" and corresponding to the surface S_1 , and also with respect to indices carrying subscript "2" and corresponding to the surface S_2 . It is easy to check, that this requirement transforms into the following relations:

$$\hat{e} \cdot \hat{1} = \hat{1} \cdot \hat{e} = \hat{e} \quad ; \quad \hat{1}^2 = \hat{1} \quad ; \quad \hat{e}^2 = \hat{1} \quad (32)$$

In case of antiparallel orientations of surfaces the matrix \hat{e} should be multiplied from the right with respect to indices carrying subscript "2" but from the left with respect to

indices carrying subscript "1", which corresponds to the fact of opposite ordering of the fields $F^{(1)}$ and $F^{(2)}$. It results in the algebra different from (32):

$$\hat{e} \cdot \hat{1} = \hat{1} \cdot \hat{e} = \hat{e} ; \quad \hat{1}^2 = \hat{1} ; \quad \hat{e}^2 = N_c \hat{e} \quad (33)$$

We also notice that in both cases $\text{Tr}_1 \text{Tr}_2 \hat{1} = N_c^2$ and $\text{Tr}_1 \text{Tr}_2 \hat{e} = N_c$.

With (32), (33) at hand, we can easily compute (27). For parallel orientation of surfaces one gets

$$\begin{aligned} \chi^{(0)}(C_1, C_2) = & \left[\frac{1}{2} \left(1 - \frac{1}{N_c} \right) \exp(\Lambda_0 + \Lambda_1 - \Lambda_e) + \right. \\ & \left. + \frac{1}{2} \left(1 + \frac{1}{N_c} \right) \exp(\Lambda_0 + \Lambda_1 + \Lambda_e) \right] - \exp(\Lambda_0^{min}) \end{aligned} \quad (34)$$

where the last term corresponds to the product of averages of two loops. For oppositely directed contours the result is

$$\chi^{(0)}(C_1, C_2) = \left[\frac{1}{N_c^2} \exp(\Lambda_0 + \Lambda_1 + N_c \Lambda_e) + \left(1 - \frac{1}{N_c^2} \right) \exp(\Lambda_0 + \Lambda_1) \right] - \exp(\Lambda_0^{min}) \quad (35)$$

Expressions (34), (35) together with the prescription for the choice of optimal integration surface provide the answer for nonperturbative interaction term. Let us come to concrete examples and first consider the simplest possible case when $S_1 = S_1^{min}$, $S_2 = S_2^{min}$ and $S_2^{min} \subset S_1^{min}$ (see Fig.1). As is was explained in details above, we always assume Gaussian dominance. One easily gets the following result for the contours with linear sizes greater than T_g and omitting perimeter contributions:

$$\Lambda_1 = \pm \frac{2}{N_c^2 - 1} \sigma S_2^{min} ; \quad \Lambda_e = \mp \frac{2N_c}{N_c^2 - 1} \sigma S_2^{min} \quad (36)$$

and (34), (35) become

$$\begin{aligned} \chi^{(0)}(C_1, C_2) = & \exp(-\sigma S_1^{min} - \sigma S_2^{min}) \cdot \left[\frac{1}{2} \left(1 - \frac{1}{N_c} \right) \exp\left(\frac{2\sigma S_2^{min}}{N_c - 1}\right) + \right. \\ & \left. + \frac{1}{2} \left(1 + \frac{1}{N_c} \right) \exp\left(-\frac{2\sigma S_2^{min}}{N_c + 1}\right) \right] - \exp(-\sigma S_1^{min} - \sigma S_2^{min}). \end{aligned} \quad (37)$$

for parallel orientations and

$$\begin{aligned} \chi^{(0)}(C_1, C_2) = & \exp(-\sigma S_1^{min} - \sigma S_2^{min}) \cdot \left[\frac{1}{N_c^2} \exp(2\sigma S_2^{min}) + \right. \\ & \left. + \left(1 - \frac{1}{N_c^2} \right) \exp\left(-\frac{2\sigma S_2^{min}}{N_c^2 - 1}\right) \right] - \exp(-\sigma S_1^{min} - \sigma S_2^{min}). \end{aligned} \quad (38)$$

for opposite orientations. It is worth reminding that $S_1^{min} > S_2^{min}$ by our convention.

Notice that in case of coinciding, but oppositely directed contours (i.e. $C = C_1 = [C_2]^\dagger$), expression (38) reproduces (17), if adjoint string tension is given by the Casimir ratio

$$\sigma_{adj} = \frac{2N_c^2}{N_c^2 - 1} \sigma \quad (39)$$

This result is to be expected since, as it was already mentioned, Gaussian dominance yields Casimir scaling [8]. If both contours C_1, C_2 lie on the same plane and C_2 is inside C_1 , the geometry becomes effectively two dimensional and the results (37), (38) coincide with formulas obtained in slightly different way in [12]. The same expressions hold true in 1+1 dimensional Yang-Mills theory, where one has just two dimensional geometry and, on the other hand, exact Gaussian picture. One also look upon (37), (38) as an algebraic rule of adding up parallel or antiparallel fundamental fluxes which illustrate the decomposition $\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$ and $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$ respectively, with the string tension in each representation given by the Casimir scaling law .

We are now interested in the case of contours separated by distances greater than T_g . By way of example let us calculate purely nonperturbative correlator of two Wilson loops $\langle W(C_1)W(C_2) \rangle$ for simple rectangular geometry of the contours. We choose two rectangular contours $R \times T$ lying on parallel planes, at distance h from each other (see Figs. 2, 3). We suppose that $T \gg R$ and will not take care of subleading $1/T$ terms. If h is of the order of T_g , one comes back to the case described by (37), (38). We take the distance h such that $R \gtrsim h \gtrsim T_g$, where the nonperturbative regime is supposed to play a role. One could still choose for S_1 and S_2 the corresponding minimal surfaces S_1^{min} and S_2^{min} . In this case the correlator $\chi^{(0)}$ would be equal to zero up to exponentially small terms of the kind $\exp(-2h/T_g)$, as it is clearly seen from (29), (30). Such configuration of surfaces however does not correspond to the minimal energy condition formulated above and is therefore unstable. It is clear that the subtraction term, the last exponent on the r.h.s. of (37), (38) stays intact, since it is a product of single loop averages. Other terms, on the contrary, strongly depend on the profile of the strings, which defines the integration surface and is to be chosen according to dynamical minimal energy condition. Consider first the case of opposite loop orientations and let us examine different choices of surfaces. As a trivial example we might adopt the same choice as above, namely $S_1 = S_1^{min}; S_2 = S_2^{min}$. The first term in square brackets in (35) contributes to $\langle W(C_1)W(C_2) \rangle$ as

$$\frac{1}{N_c^2} \exp(-2\sigma RT)$$

as it should be since $\Lambda_0 = \Lambda_0^{min} = \sigma S_1^{min} + \sigma S_2^{min} = 2\sigma RT$ and $\Lambda_1 = \Lambda_e = 0$ for this choice of surfaces (up to exponentially small terms $\sim \exp(-2h/T_g)$, which we always omit in the paper). Correspondingly, the second term in square brackets in (35) would be

$$\left(1 - \frac{1}{N_c^2}\right) \exp(-2\sigma RT)$$

with the sum of two giving expected answer $\langle W(C_1) \rangle \langle W(C_2) \rangle = \exp(-2\sigma RT)$ and hence $\chi^{(0)} = 0$. One immediately sees that such choice is not optimal if h is small. Instead, if we choose S_1 as a minimal enveloping surface with boundary on C_1 and coinciding with $S_2 = S_2^{min}$ inside C_2 (due to apparent symmetry of our problem, one could of course easily interchange indices '1' and '2'), we get for the first term in square brackets in (35)

$$\frac{1}{N_c^2} \exp(-2\sigma hT)$$

where $2hT = S_{12} = S_1 - S_2$. The second term contributes to $\langle W(C_1)W(C_2) \rangle$ as

$$\left(1 - \frac{1}{N_c^2}\right) \exp(-2\sigma hT - \sigma_{adj} RT)$$

where σ_{adj} is given by (39) and this contribution is always subleading with respect to the former one if $R \gg h$ and N_c is not exponentially large (see below). So these two different choices give different answers for Wilson loop correlator:

$$\langle W(C_1)W(C_2) \rangle = \exp(-2\sigma RT) \quad (40)$$

in the first case (two individual minimal surfaces) and

$$\langle W(C_1)W(C_2) \rangle = \frac{1}{N_c^2} \exp(-2\sigma hT) + \left(1 - \frac{1}{N_c^2}\right) \exp(-2\sigma hT - \sigma_{adj} RT) \quad (41)$$

in the second case (enveloping geometry). According to our criteria, the answer which is dominant should be chosen as correct, since it corresponds to actual string configuration. It is seen, that there is a critical distance between loops

$$h_{crit} \approx R - \frac{1}{\sigma T} \log N_c \quad (42)$$

in our problem.⁵ For $h < h_{crit}$ confining strings rearrange themselves⁶ with respect to noninteracting case, which is encoded in expression (41). Correspondingly, one has nonzero $\chi^{(0)}$. For larger h they do not interact and $\chi^{(0)}(C_1, C_2)$ vanishes. It is important that in our picture it happens dynamically, in particular, one cannot just naively take large N_c -limit in (34), (35). On the other hand, if h is kept fixed, then it is clear from (42) that in large N_c limit $\chi^{(0)}$ should vanish. It is worth saying a few words about the meaning of T in (40), (41). It is seen that with T going to infinity, h_{crit} is increasing and approaching R . Therefore for long static loops minimal energy state will always eventually win, as it should be. If however one studies not correlators of static loops but some physical process like hadron scattering, pre-exponent factors are important and T corresponds to the typical interaction time. In particular cases it can be quite small.

⁵For abelian confining strings including soap films the last term in (42) is absent.

⁶See also [13], where static multiquark interactions were studied in strong coupling expansion regime.

Expression (42) then physically means that if T is small, strings does not have enough time to rearrange and nonperturbative interaction does not take place.

To summarize, the answer is given by

$$\chi^{(0)} = \frac{1}{N_c^2} \exp(-2\sigma hT) + \left(1 - \frac{1}{N_c^2}\right) \exp(-2\sigma hT - \sigma_{adj}RT) - \exp(-2\sigma RT) \quad (43)$$

if $h < h_{crit}$ and $\chi^{(0)} = 0$ if $h > h_{crit}$. We now come to the case of parallel orientations. Let us consider two geometries analyzed above. The case of individual minimal surfaces is identical for parallel and opposite orientations and has just been considered. The enveloping geometry gives for the first term in square brackets in (34)

$$\frac{1}{2} \left(1 - \frac{1}{N_c}\right) \exp\left(-2\sigma hT - \frac{2\sigma RT(N_c - 2)}{N_c - 1}\right)$$

while the second term becomes

$$\frac{1}{2} \left(1 + \frac{1}{N_c}\right) \exp\left(-2\sigma hT - \frac{2\sigma RT(N_c + 2)}{N_c + 1}\right)$$

and is always subleading. In particular case $N_c = 3$ the former term corresponds to the creation of the string in $\bar{\mathbf{3}}$ representation of the length R which has the same tension as the fundamental string while the second term describes sextet representation string formation $\mathbf{6} = \text{Symm}\{\mathbf{3} \otimes \mathbf{3}\}$ with tension $2(N_c + 2)/(N_c + 1) = 2.5$ times larger than fundamental one. In this particular case of $N_c = 3$ one can additionally consider double Y-shape profile, shown on Fig.3, with the result for the first term in square brackets in (34)

$$\frac{1}{3} \exp(-\sqrt{3}\sigma hT - \sigma RT)$$

Since $\sqrt{3} < 2$, this term is seen to be dominant over enveloping geometry in $N_c = 3$ case.

As in opposite orientations case there appears critical length h_{crit} which for $N_c = 3$ is given by

$$h_{crit} = \frac{1}{\sqrt{3}} \left(R - \frac{\log 3}{\sigma T}\right) \quad (44)$$

The expression for $\chi^{(0)}$ becomes ($N_c = 3$):

$$\begin{aligned} \chi^{(0)}(C_1, C_2) = \exp(-\sigma S_{12}) \cdot \left[\frac{1}{3} \exp(-\sigma S_3) + \right. \\ \left. + \frac{2}{3} \exp\left(-\frac{5\sigma S_3}{2}\right) \right] - \exp(-2\sigma RT) \end{aligned} \quad (45)$$

where S_{12} represents the boundary surface $S_{12} = 4hT/\sqrt{3}$ and $S_3 = (R - h/\sqrt{3})T$ is the common part of S_1 and S_2 . Notice that $l_{crit} = R - h_{crit}/\sqrt{3} > 0$, therefore one is never

in the situation of S_3 shrinking to zero. In some sense l_{crit} in (45) plays the role of R_{min} from (16).

The physical picture is the same as for opposite orientations – at small distances h , $h_{crit} > h \gtrsim T_g$ preferable string configuration is given by double Y-shape profile shown on Fig.3 while at larger distances there is no common string state formation and hence nonperturbative interaction is absent, $\chi^{(0)}(C_1, C_2) = 0$.

It is important to compare at this point our approach to another approach, which is also based on field correlator method, but follows another logic. This well-known strategy started in [9] has many successful phenomenological applications and modifications (see, e.g. [14]). There are two essential differences to be mentioned. The first one comes from the fact that we are working in Euclidean metric rather than in Minkowski. It is important to stress, that in our approach the vacuum averaging over gluon fields (which eventually creates the string) is to be done in Euclidean space with Euclidean gluon configurations, and transition to Minkowski space can be done only after this vacuum averaging. This is intimately connected to the realization that vacuum configurations are of tunnelling type and it is not legitimate to continue them analytically into Minkowski space. However, as soon as the fields are integrated out the transition to the latter is not a problem and as we demonstrated in [15] the QCD string is present also on the light cone. Correspondingly, we do not have a notion of light-like surfaces with vanishing areas, which is of importance in the formalism developed in [9]. The second point is a different physical picture behind the purely nonperturbative interaction in our model. The magnitude of $\chi^{(0)}(C_1, C_2)$ depends in our approach on the chosen string worldsheet configuration which, in its turn, is determined by the dynamical minimal energy requirement, discussed above. However, we do not study effects of string overlap, caused by finite thickness of the strings. If the minimal surface requirement dictates $S_{1,2} = S_{1,2}^{min}$, which always happen for well separated loops, there is no purely nonperturbative interaction in our picture, i.e. $\chi^{(0)}(C_1, C_2) = 0$. This is in sharp contrast to the approach [9] where all physical effect is due to nonzero value of T_g which bring about nonzero overlap between hadron Wilson loops. In other words, the expressions (37), (38) have nontrivial limit if $T_g \rightarrow 0$, while string tension $\sigma \sim \langle F^2 \rangle T_g^2$ is kept fixed, contrary to the corresponding expressions from [9, 14] which vanish in this limit. Accordingly the hadron–hadron cross-section in that approach is proportional to T_g in the tenth power [14] and is very sensitive to exact value of T_g . As it is, both approaches are viable in their regions of values of T_g , and it is very important to extract this value from lattice data, analytically or phenomenologically. The existing calculations yield values in the range $0.35 \text{ Fm} > T_g > 0.1 \text{ Fm}$ for different lattice and analytic procedures (see [16] and references therein). We assume in our approach that T_g is closer to lower limit and therefore the string overlap effect proportional to higher powers of T_g is a small correction to our basic mechanism. One should stress however, that if even T_g would be large to justify approach of [9, 14] at moderate energies, the gluon-exchange-generated Pomeron mechanism is always dominating asymptotically at large energies.

To conclude this section let us say a few words about the case of distant loops. The general expression (22) is applicable here as well. In Gaussian approximation one still has (27). Important change however is that now correlator $\langle F^{(1)} F^{(2)} \rangle$ does not get any contributions since the surfaces S_1, S_2 should be chosen according to our principle, as independent and not intersecting minimal surfaces. Hence nonperturbative contribution vanishes in this case, up to the string overlap effects mentioned above.

In other words when contours are distant from each other, it is always preferable to deform S_{12} to two discs, corresponding to minimal surfaces for each contour plus whatever thin tube connecting these surfaces through the point x_0 , thus reducing $\chi^{(0)}(C_1, C_2)$ to zero. This is a sign, that purely nonperturbative contribution vanishes, and one should consider next terms in the expansion (8), namely $\chi^{(4)}(C_1, C_2)$. It is important to realize that the surface entering (9) is not dynamical⁷, which reflects itself in the possibility of infinite squeezing of the tube, connecting two distant minimal surfaces. The situation changes however when one includes perturbative gluons, propagating inside its wall and forming a physical glueball state in this way. It is actually this glueball exchange mechanism, which corresponds to the term $\chi^{(4)}(C_1, C_2)$ when background field is taken into account. We study this term in the next section.

4 Glueball exchange interaction

We now turn to the term $\chi^{(4)}(C_1, C_2)$ in the expansion (8). We shall see that in the absence of NP background this term reduces to the purely perturbative two-gluon exchange term suggested in [17] as a basic element of Pomeron exchange. To be more precise, let us consider first the case of no background fields. In the Feynman gauge for a_μ , one has in the lowest order

$$\langle a_\mu^a(x) a_\nu^b(y) \rangle = \delta_{\mu\nu} \delta^{ab} \frac{1}{4\pi^2(x-y)^2} \quad (46)$$

Our expression for $\chi^{(4)}$ will have the same form as (27), where one should keep only nonperturbative background field B_μ in diagonal correlators $\langle \text{Tr } F^1 F^1 \rangle$ while terms proportional to $\langle \text{Tr } F^1 F^2 \rangle$ contain only perturbative exchange. These perturbative exchanges are modified however by the presence of nonperturbative background. To take it into account, one has to perform averaging in two steps: first in valence (perturbative) field a_μ and second - in background field B_μ :

$$\begin{aligned} \langle W(C_1) W(C_2) \rangle &= \frac{g^4}{N_c^2} \text{Tr}_1 \text{Tr}_2 P_{12} \int dx_{\mu_1}^{(1)} \int dx_{\mu_2}^{(2)} \int dy_{\nu_1}^{(1)} \int dy_{\nu_2}^{(2)} \\ &\Phi_{C_1}(x^{(2)}, x^{(1)}) t^{a_1} \Phi_{C_1}(x^{(1)}, x^{(2)}) t^{a_2} \Phi_{C_2}(y^{(2)}, y^{(1)}) t^{b_1} \Phi_{C_2}(y^{(1)}, y^{(2)}) t^{b_2}. \end{aligned}$$

⁷It has come from the Stokes theorem and therefore could be arbitrary, subject to our principle of minimal action.

$$\cdot (\langle a(x^{(1)})a(y^{(1)}) \rangle \langle a(x^{(2)})a(y^{(2)}) \rangle + \langle a(x^{(1)})a(y^{(2)}) \rangle \langle a(x^{(2)})a(y^{(1)}) \rangle) \quad (47)$$

where short-hand notation was used $a(x^{(1)}) \equiv a_{\mu_1}^{a_1}(x^{(1)})$. The coordinates $x^{(1)}$, $x^{(2)}$ are ordered along the contour C_1 as well as $y^{(1)}$, $y^{(2)}$ are ordered along the contour C_2 . Notice also gauge-invariance of (47) due to transformation law (2).

Before proceeding further one is to define the dependence of $\langle a_\mu(x)a_\nu(y) \rangle$ on background fields. To this end it is convenient to use Feynman-Schwinger representation for gluon Green's function [3] and represent it as

$$G_{\mu\nu}^{ab} = \langle a_\mu^a(x)a_\nu^b(y) \rangle = \int_0^\infty ds (\mathcal{D}z)_{xy} \exp(-K_0) \Phi_{\mu\nu}^{ab}(x, y) \quad (48)$$

where

$$K_0 = \frac{1}{4} \int_0^\infty d\tau \left(\frac{dz_\mu}{d\tau} \right)^2$$

and

$$\Phi_{\mu\nu}^{ab}(x, y) = \left[P_F P_A \exp \left(ig \int_y^x A_\mu(z) dz_\mu \right) \exp \left(2g \int_0^s d\tau F(z(\tau)) \right) \right]_{\mu\nu}^{ab}$$

and a, b are adjoint color indices, whereas μ, ν - Lorentz indices, i.e. $[F]_{\mu\nu}^{ab} = -iF_{\mu\nu}^c f^{abc}$.

To understand better the topology of the resulting construction, it is useful to consider large N_c limit. One can write in this limit for adjoint phase factors in (48)

$$\begin{aligned} [t^a]_{\alpha\beta} \Phi^{ab}(x, y) [t^b]_{\gamma\delta} &= P_A \exp \left(ig \int_y^x \hat{A}_\lambda dz_\lambda \right)^{ab} [t^a]_{\alpha\beta} [t^b]_{\gamma\delta} = \\ &= \Phi_{\alpha\delta}(x, y) \Phi_{\gamma\beta}(y, x) + \mathcal{O} \left(\frac{1}{N_c} \right) \end{aligned} \quad (49)$$

where $\Phi_{\alpha\delta}(x, y)$ is parallel transporter in fundamental representation. Expression (49) exemplifies well known 't Hooft's rule for replacing gluon line by double adjoint in large- N_c limit. Inserting (49) into (47), one obviously obtains two new Wilson loops C_{12} and C_{12}' instead of previous C_1 and C_2 : each initial loop is now divided by two gluon emissions/absorptions into two arcs which are connected by double lines of gluon propagators (see Figs. 4,5). At small N_c this construction goes over into that of two fundamental loops C_1 and C_2 connected by two adjoint lines and final result will amount to replacing double fundamental string worldsheet by one adjoint string worldsheet. In terms of string tensions it correspond to replacement of 2σ by $9\sigma/4$ in Gaussian approximation. We will keep large N_c limit and replacement (49) in what follows.

The averaging over background fields leads to the following result:

$$\langle \langle W(C_1)W(C_2) \rangle_a^{(4)} \rangle_B = \chi^{(4)}(C_1, C_2) =$$

$$= \frac{\tilde{g}^4}{N_c^2} \int_{L_{12}} ds_1 \mathcal{D}z^{(1)} \int_{L_{12}'} ds_2 \mathcal{D}z^{(2)} \exp(-K_0^{(1)} - K_0^{(2)}) \langle W(C_{12})W(C_{12}') \rangle \quad (50)$$

where $\tilde{g}^2 = g^2 N_c$. Here contours C_{12} and C_{12}' comprise pieces of C_1 and C_2 connected by two double fundamental lines L_{12} and L_{12}' . It is understood that surfaces S_{12} , S_{12}' are subjects of our general assumption about minimal action. This gives different forms depending on the distance between original loops C_1 and C_2 , see below. It is also understood that gluon spin operators $2gF(z)$ are to be placed on the gluon trajectories L_{12} and L_{12}' in accordance with (48). It will produce gluon spin interaction terms which influence glueball Green's function; to simplify discussion we omit these terms at the moment.

Now we can use large- N_c factorization property for the product $\langle W(C_{12})W(C_{12}') \rangle$ and use area law asymptotics for each piece, i.e. for surfaces S_{12} and S_{12}' . One obtains

$$\chi^{(4)}(C_1, C_2) = \frac{\tilde{g}^4}{N_c^2} \int_{L_{12}} ds_1 \mathcal{D}z^{(1)} \int_{L_{12}'} ds_2 \mathcal{D}z^{(2)} \exp(-K_0^{(1)} - K_0^{(2)}) \exp(-\sigma(S_{12} + S_{12}')) \quad (51)$$

To define the profiles of the surfaces we shall use the same principle outlined above, i.e. we require an effective value of area $\langle S_{12} + S_{12}' \rangle$ averaged over possible gluon trajectories L_{12} and L_{12}' to be minimal. The result will of course strongly depend on relative positions and orientations of the contours C_1 and C_2 . In the first case when both loops lie on the same plane and C_2 is entirely inside C_1 , it is clear that the sum $S_{12} + S_{12}'$ does not depend on trajectories L_{12} and L_{12}' , and one has $S_{12} + S_{12}' = S_1 - S_2$. Thus one obtains effectively the surface S_1 with the hole due to C_2 , i.e. a construction which already has appeared in purely nonperturbative term for opposite oriented contours C_1, C_2 , but now with two valence gluons connecting contours C_1 and C_2 , see Fig.4.

However for large enough distances the true minimum of $S_{12} + S_{12}'$ is reached by another construction – when two contours C_1 and C_2 are connected by narrow strip formed by trajectories L_{12} and L_{12}' with the double (adjoint) string worldsheet between them (Fig.5). This narrow strip is nothing but the glueball Green's function and the width of the strip is equal to the average size of the lowest mass glueball, i.e. around 0.5 Fm. Notice that due to kinetic terms in (51) this strip is dynamical (contrary to the nonperturbative case) and cannot be shrunk. This is a typical construction for high-energy scattering amplitude when the glueball exchange diagram is gradually replaced by glueball Regge trajectory exchange, i.e. by Pomeron exchange, which persists to larger experimentally accessible energies.

To demonstrate that explicitly, one should rewrite expression (51) directly in terms of glueball Green's function:

$$\frac{\chi^{(4)}(C_1, C_2)}{\langle W(C_1)W(C_2) \rangle} = P_{12} \int_{C_1} dx_{\mu_1}^{(1)} \int_{C_1} dx_{\mu_2}^{(2)} \int_{C_2} dy_{\nu_1}^{(1)} \int_{C_2} dy_{\nu_2}^{(2)} \left[G_{\mu_1 \mu_2}^{\nu_1 \nu_2}(x^{(1)}, x^{(2)} | y^{(1)}, y^{(2)}) + (y^{(1)} \leftrightarrow y^{(2)}) \right] \quad (52)$$

where $G_{\mu_1\mu_2}^{\nu_1\nu_2}(x^{(1)}, x^{(2)}|y^{(1)}, y^{(2)})$ is two-gluon glueball Green's function, describing propagation from points $x^{(1)}, x^{(2)}$ to $y^{(1)}, y^{(2)}$, which has the Feynman-Schwinger representation as in (51). The spectrum of this Green's function (with spin terms included) was computed analytically in [18].

In case all points are close to each other, i.e.

$$|x^{(i)} - x^{(j)}| \ll T_g \quad ; \quad |x^{(i)} - y^{(j)}| \ll T_g$$

one can replace glueball Green's function by a product of free gluon propagators

$$G_{\mu_1\mu_2}^{\nu_1\nu_2}(x^{(1)}, x^{(2)}|y^{(1)}, y^{(2)}) \sim g^4 \frac{\delta_{\mu_1\nu_1}\delta_{\mu_2\nu_2}}{(2\pi)^4(x^{(1)} - y^{(1)})^2(x^{(2)} - y^{(2)})^2} \quad (53)$$

Another asymptotics is available when both $|x^{(i)} - y^{(j)}|$ are large, then the spectral decomposition is possible:

$$\begin{aligned} & G_{\mu_1\mu_2}^{\nu_1\nu_2}(x^{(1)}, x^{(2)}|y^{(1)}, y^{(2)}) \sim \\ & \sim \sum_n \Psi_{\mu_1\mu_2}^{(n)}(x^{(1)}, x^{(2)}) \Psi_{\nu_1\nu_2}^{(n)\dagger}(y^{(1)}, y^{(2)}) \cdot \exp\left(-M_n \cdot \left|\frac{x^{(1)} + x^{(2)}}{2} - \frac{y^{(1)} + y^{(2)}}{2}\right|\right) \end{aligned} \quad (54)$$

Since the lowest glueball is rather heavy, $M_0 \approx 1.5$ GeV, one expect fast decrease of $\chi^{(4)}$ when distance between loops is growing:

$$\chi^{(4)}(h) \sim \exp(-M_0|h|)$$

The situation is qualitatively similar to the one studied in [19, 20], where gluon was assumed to have effective mass $m_g \sim 0.9$ GeV. One expects dipole-dipole cross section around a few mb in this case, when α_s is of the order of one. To obtain realistic large hadron-hadron scattering one needs glueball exchange to be reggeized, in which case radius of interaction grows logarithmically [21]. In particular case of BFKL Pomeron this picture was studied in [22]. Our picture differs from that of BFKL, since nonperturbative background is taken into account. Let us consider as an example the problem of high-energy forward onium-onium scattering. Since the interaction time between particles at high energies is much smaller than the typical interaction time for quarks inside onium, one can consider the onium in this process as free quark-antiquark pair (see, e.g. [19]). The small radius of the onium compared with typical transversal length scales of the problem dictates $\chi^{(4)}$ -dominance over $\chi^{(0)}$ in the problem since possible nonperturbative string configurations which could contribute to $\chi^{(0)}$ are strongly suppressed over individual minimal noninteracting strings.

Since it is more convenient to study the scattering of systems in given quantum states rather than scattering of Wilson loops, we are to switch from (15) to spectral decomposition of the form (13) and take only one term, corresponding to the scattering of particular states. The resulting expression coincides (up to normalization factor) with

the scattering amplitude (see, e.g. [23]). Since in the Feynman gauge for the field a_μ^a we have

$$G_{\mu_1\mu_2}^{\nu_1\nu_2}(x^{(1)}, x^{(2)}|y^{(1)}, y^{(2)}) = \delta_{\mu_1\nu_1}\delta_{\mu_2\nu_2}G^{(2)}(x^{(1)}, x^{(2)}|y^{(1)}, y^{(2)}) \quad (55)$$

the answer can be straightforwardly obtained using the same strategy as in [19]:

$$T_{forw} = i \int d^2\rho_1 \int d^2\rho_2 \int_0^1 dz_1 \int_0^1 dz_2 |\psi(\rho_1, z_1)|^2 |\psi(\rho_2, z_2)|^2 F(\rho_1, \rho_2, Q=0) \quad (56)$$

where

$$F(\rho_1, \rho_2, Q=0) = \frac{N_c^2 - 1}{32\pi^2 N_c^2} \int d^2k G^{(2)}(k, Q=0) \cdot (2 - \exp(-ik\rho_1) - \exp(ik\rho_1))(2 - \exp(-ik\rho_2) - \exp(ik\rho_2)) \quad (57)$$

In the above expression $G^{(2)}(k, Q)$ is Fourier transform of (55) with respect to total momentum $p_1 + p_2 = Q$ and relative momentum $p_1 - p_2 = Q - 2k$, the former is equal to zero for forward scattering amplitude (we also suppose in (57) vanishing transverse momenta of onia). The mixed representation wave function $\psi(\rho_1, z_1)$ defined on the light-cone describes the state of color dipole with transverse size ρ_1 and fraction of total onium light-cone momentum p_1^+ carried by quark z_1 . We omit spinor indices, assuming proper summation over them. If one will "turn off" confinement (i.e. in our formalism put the confining background field to zero everywhere), the Green's function in the leading order of perturbation theory would be just a product of two gluon propagators

$$G_{free}^{(2)}(k, Q) = \frac{g^4}{(Q-k)^2 k^2} \quad (58)$$

and inserting (58) into (57) one returns to the results of [19]. One would expect that the effects of confinement suppress amplitude [19] in two different ways: first, because of the mass gap (and actually quite large mass even of the lightest glueball), and, second, due to the glueball wave function fast decrease at large relative distances (see (54)). These properties solve the artefact of color Van der Waals forces, appearing in purely perturbative dipole-dipole interaction.

5 Conclusions

In the present paper we discussed interactions of Wilson loops in confining theory, having in mind gluodynamics as a concrete example. The effects of confinement were taken into account in the formalism of perturbation theory in confining background. We have described the background by gauge-invariant Gaussian correlator with small correlation length, which is supported by lattice and analytic calculations. Two main physically different mechanisms of interaction were analyzed. The first one, which we call nonperturbative, refers to the process of confining string rearrangement, which can be energetically

preferable for particular geometries of the contours. In this way a common surface of two contours C_1, C_2 is created and in case of opposite orientation this surface is a ring between C_1 and C_2 (with a hole inside the smaller loop). This mechanism has a direct classical analog in soap films, while for parallel orientation nonabelian properties of Wilson loops lead to a nonclassical configuration with the same ring but the hole filled by the film. The second mechanism arises due to two-gluon exchange between loops and the corresponding amplitude is $O(g^4)$. In the confining background and at large N_c this simple picture of two contours connected by two gluon lines is transformed into a new geometry of two new composite loops, as shown in Figs.4,5. As a result one has two types of surface configurations – for small and for large separations between minimal surfaces S_1 and S_2 , shown respectively in Fig.4 and Fig.5. It is argued in the paper that the configuration generic for the scattering corresponds to the Fig.5, and reduces to the (reggeized) glueball exchanges between loops while for the case of decay and sea quark loop effects both nonperturbative and perturbative mechanisms are important with small separation between S_1 and S_2 . Results obtained in the paper provide a basis for a systematic development both in direction of nonperturbative approach to the hadron scattering and in the direction of the theory of strong hadron decays.

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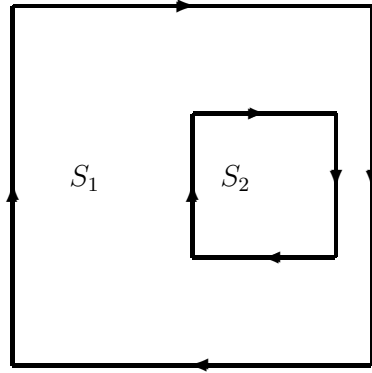


Figure 1: Planar geometry of the Wilson loops for $S_2^{min} \subset S_1^{min}$ (the case of coinciding orientations).

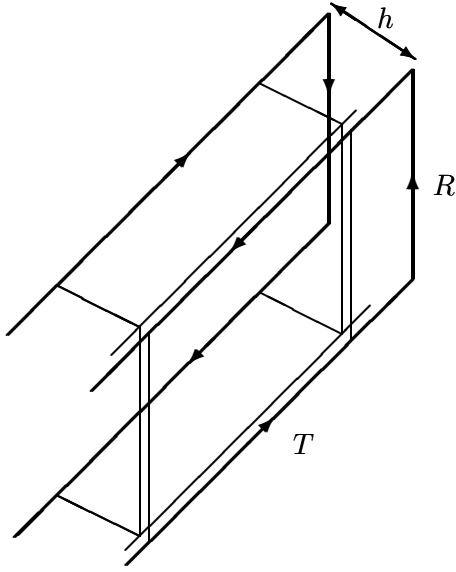


Figure 2: Nonperturbative interaction of rectangular Wilson contours. The minimal string profile for the case of opposite orientations is given by "enveloping" geometry. Leading large area contribution corresponds to annihilation of fluxes along R .

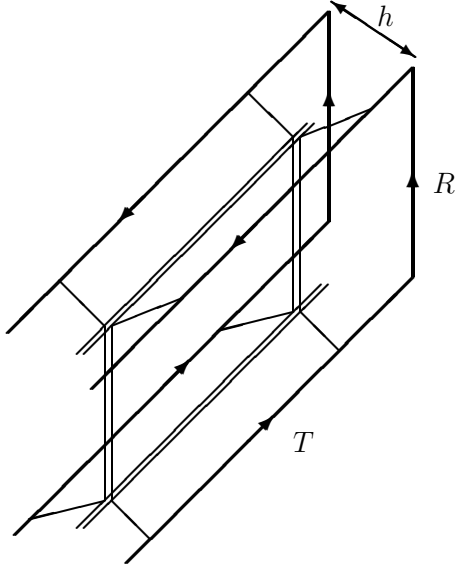


Figure 3: Nonperturbative interaction of rectangular Wilson contours. The "time-slices" of the minimal string profile for the case of parallel orientations are depicted. It is assumed that $N_c = 3$. Leading large area contribution corresponds to a single fundamental flux in R -direction.

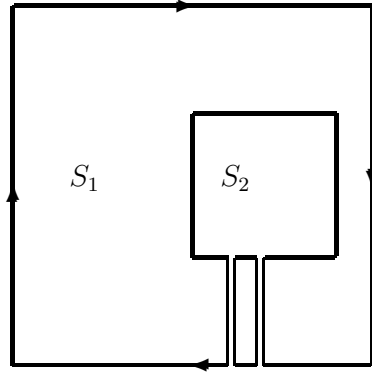


Figure 4: The same as Figure 5, but with the two-dimensional geometry of Figure 1.

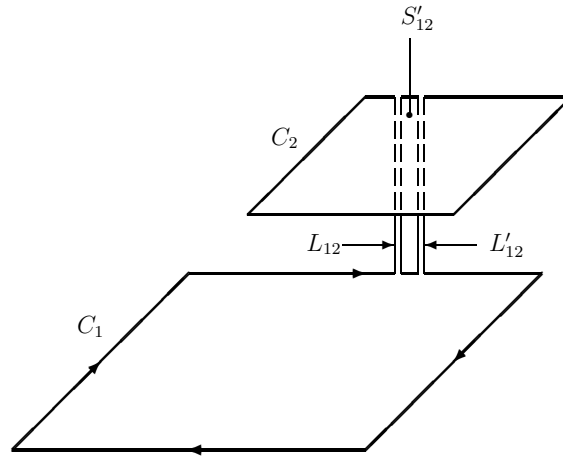


Figure 5: Dominant two-gluon glueball term responsible for perturbative interaction in nonperturbative background at large distances. Gluon propagator lines are replaced by double fundamental lines in large N_c limit.